Appendix

Texas Essential Knowledge and Skills

Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter C. High School

Statutory Authority: The provisions of this Subchapter C issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

The provisions of this subchapter shall be implemented beginning September 1, 1998, and at that time, shall supersede §75.63(e)-(g) of this title (relating to Mathematics).

Source: The provisions of this §111.31 adopted to be effective September 1, 1996, 21 TexReg 7371.

§111.32. Algebra I (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students will continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students use symbols in a variety of ways to study relationships among quantities.

(3) Function concepts. Functions represent the systematic dependence of one quantity on another. Students use functions to represent and model problem situations and to analyze and interpret relationships.

(4) Relationship between equations and functions. Equations arise as a way of asking and answering questions involving functional relationships. Students work in many situations to set up equations and use a variety of methods to solve these equations.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(b) **Foundations for functions**: knowledge and skills and performance descriptions.

(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Following are performance descriptions.

(A) The student describes independent and dependent quantities in functional relationships.

(B) The student gathers and records data, or uses data sets, to determine functional (systematic) relationships between quantities.

(C) The student describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations.

(D) The student represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(E) The student interprets and makes inferences from functional relationships.

(2) The student uses the properties and attributes of functions.

Following are performance descriptions.

(A) The student identifies and sketches the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions.

(B) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(C) The student interprets situations in terms of given graphs or creates situations that fit given graphs.

(D) In solving problems, the student collects and organizes data, makes and interprets scatterplots, and models, predicts, and makes decisions and critical judgments.

(3) The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

Following are performance descriptions.

(A) The student uses symbols to represent unknowns and variables.

(B) Given situations, the student looks for patterns and represents generalizations algebraically.

(4) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

Following are performance descriptions.

(A) The student finds specific function values, simplifies polynomial expressions, transforms and solves equations, and factors as necessary in problem situations.

(B) The student uses the commutative, associative, and distributive properties to simplify algebraic expressions.

(c) **Linear functions**: knowledge and skills and performance descriptions.

(1) The student understands that linear functions can be represented in different ways and translates among their various representations.

Following are performance descriptions.

(A) The student determines whether or not given situations can be represented by linear functions.

(B) The student determines the domain and range values for which linear functions make sense for given situations.

(C) The student translates among and uses algebraic, tabular, graphical, or verbal descriptions of linear functions.

(2) The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

Following are performance descriptions.

(A) The student develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations.

(B) The student interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

(C) The student investigates, describes, and predicts the effects of changes in m and b on the graph of $y = mx + b$.

(D) The student graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept.

(E) The student determines the intercepts of linear functions from graphs, tables, and algebraic representations.

(F) The student interprets and predicts the effects of changing slope and y-intercept in applied situations.

(G) The student relates direct variation to linear functions and solves problems involving proportional change.

(3) The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Following are performance descriptions.

(A) The student analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems.

(B) The student investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

(C) For given contexts, the student interprets and determines the reasonableness of solutions to linear equations and inequalities.

(4) The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Following are performance descriptions.

(A) The student analyzes situations and formulates systems of linear equations to solve problems.

(B) The student solves systems of linear equations using concrete models, graphs, tables, and algebraic methods.

(C) For given contexts, the student interprets and determines the reasonableness of solutions to systems of linear equations.

(d) **Quadratic and other nonlinear functions**: knowledge and skills and performance descriptions.

(1)	The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.	Following are performance descriptions. (A) The student determines the domain and range values for which quadratic functions make sense for given situations. (B) The student investigates, describes, and predicts the effects of changes in a on the graph of $y = ax^2$. (C) The student investigates, describes, and predicts the effects of changes in c on the graph of $y = x^2 + c$. (D) For problem situations, the student analyzes graphs of quadratic functions and draws conclusions.
(2)	The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.	Following are performance descriptions. (A) The student solves quadratic equations using concrete models, tables, graphs, and algebraic methods. (B) The student relates the solutions of quadratic equations to the roots of their functions.
(3)	The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.	Following are performance descriptions. (A) The student uses patterns to generate the laws of exponents and applies them in problem-solving situations. (B) The student analyzes data and represents situations involving inverse variation using concrete models, tables, graphs, or algebraic methods. (C) The student analyzes data and represents situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Source: The provisions of this §111.32 adopted to be effective September 1, 1996, 21 TexReg 7371.

§111.33. Algebra II (One-Half to One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(b) **Foundations for functions**: knowledge and skills and performance descriptions.

(1) The student uses properties and attributes of functions and applies functions to problem situations.

Following are performance descriptions.

(A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

(2) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

Following are performance descriptions.

(A) The student uses tools including matrices, factoring, and properties of exponents to simplify expressions and transform and solve equations.

(B) The student uses complex numbers to describe the solutions of quadratic equations.

(C) The student connects the function notation of y = and f(x) =.

(3) The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.

Following are performance descriptions.

(A) The student analyzes situations and formulates systems of equations or inequalities in two or more unknowns to solve problems.

(B) The student uses algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

(C) For given contexts, the student interprets and determines the reasonableness of solutions to systems of equations or inequalities.

(c) **Algebra and geometry**: knowledge and skills and performance descriptions.

(1) The student connects algebraic and geometric representations of functions.

Following are performance descriptions.

(A) The student identifies and sketches graphs of parent functions, including linear ($y = x$), quadratic ($y = x^2$), square root ($y = \sqrt{x}$), inverse ($y = 1/x$), exponential ($y = a^x$), and logarithmic ($y = \log_a x$) functions.

(B) The student extends parent functions with parameters such as m in $y = mx$ and describes parameter changes on the graph of parent functions.

(C) The student recognizes inverse relationships between various functions.

(2) The student knows the relationship between the geometric and algebraic descriptions of conic sections.

Following are performance descriptions.

(A) The student describes a conic section as the intersection of a plane and a cone.

(B) In order to sketch graphs of conic sections, the student relates simple parameter changes in the equation to corresponding changes in the graph.

(C) The student identifies symmetries from graphs of conic sections.

(D) The student identifies the conic section from a given equation.

(E) The student uses the method of completing the square.

(d) **Quadratic and square root functions**: knowledge and skills and performance descriptions.

(1) The student understands that quadratic functions can be represented in different ways and translates among their various representations.

Following are performance descriptions.

(A) For given contexts, the student determines the reasonable domain and range values of quadratic functions, as well as interprets and determines the reasonableness of solutions to quadratic equations and inequalities.

(B) The student relates representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.

		(C) The student determines a quadratic function from its roots or a graph.
(2)	The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.	Following are performance descriptions. (A) The student uses characteristics of the quadratic parent function to sketch the related graphs and connects between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions. (B) The student uses the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.
(3)	The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	Following are performance descriptions. (A) The student analyzes situations involving quadratic functions and formulates quadratic equations or inequalities to solve problems. (B) The student analyzes and interprets the solutions of quadratic equations using discriminants and solves quadratic equations using the quadratic formula. (C) The student compares and translates between algebraic and graphical solutions of quadratic equations. (D) The student solves quadratic equations and inequalities.
(4)	The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	Following are performance descriptions. (A) The student uses the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describes limitations on the domains and ranges. (B) The student relates representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions. (C) For given contexts, the student determines the reasonable domain and range values of square root functions, as well as interprets and determines the reasonableness of solutions to square root equations and inequalities. (D) The student solves square root equations and inequalities using graphs, tables, and algebraic methods. (E) The student analyzes situations modeled by square root functions, formulates equations or inequalities, selects a method, and solves problems. (F) The student expresses inverses of quadratic functions using square root functions.

(e) **Rational functions**: knowledge and skills and performance descriptions.

The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Following are performance descriptions.

(1) The student uses quotients to describe the graphs of rational functions, describes limitations on the domains and ranges, and examines asymptotic behavior.

(2) The student analyzes various representations of rational functions with respect to problem situations.

(3) For given contexts, the student determines the reasonable domain and range values of rational functions, as well as interprets and determines the reasonableness of solutions to rational equations and inequalities.

(4) The student solves rational equations and inequalities using graphs, tables, and algebraic methods.

(5) The student analyzes a situation modeled by a rational function, formulates an equation or inequality composed of a linear or quadratic function, and solves the problem.

(6) The student uses direct and inverse variation functions as models to make predictions in problem situations.

(f) **Exponential and logarithmic functions**: knowledge and skills and performance descriptions.

The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Following are performance descriptions.

(1) The student develops the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses.

(2) The student uses the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describes limitations on the domains and ranges, and examines asymptotic behavior.

(3) For given contexts, the student determines the reasonable domain and range values of exponential and logarithmic functions, as well as interprets and determines the reasonableness of solutions to exponential and logarithmic equations and inequalities.

(4) The student solves exponential and logarithmic equations and inequalities using graphs, tables, and algebraic methods.

(5) The student analyzes a situation modeled by an exponential function, formulates an equation or inequality, and solves the problem.

111.34. Geometry (One Credit).

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; shapes and figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, algebraic, and coordinate), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities to solve meaningful problems by representing figures, transforming figures, analyzing relationships, and proving things about them.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(b) **Geometric structure**: knowledge and skills and performance descriptions.

(1) The student understands the structure of, and relationships within, an axiomatic system.	Following are performance descriptions. (A) The student develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems. (B) Through the historical development of geometric systems, the student recognizes that mathematics is developed for a variety of purposes. (C) The student compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries.
(2) The student analyzes geometric relationships in order to make and verify conjectures.	Following are performance descriptions. (A) The student uses constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.

(B)	The student makes and verifies conjectures about angles, lines, polygons, circles, and three-dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.	
	(3) The student understands the importance of logical reasoning, justification, and proof in mathematics.	Following are performance descriptions. (A) The student determines if the converse of a conditional statement is true or false. (B) The student constructs and justifies statements about geometric figures and their properties. (C) The student demonstrates what it means to prove mathematically that statements are true. (D) The student uses inductive reasoning to formulate a conjecture. (E) The student uses deductive reasoning to prove a statement.
	(4) The student uses a variety of representations to describe geometric relationships and solve problems.	Following is a performance description. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(c) **Geometric patterns**: knowledge and skills and performance descriptions.

The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures.	Following are performance descriptions. (1) The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles. (2) The student uses properties of transformations and their compositions to make connections between mathematics and the real world in applications such as tessellations or fractals. (3) The student identifies and applies patterns from right triangles to solve problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(d) **Dimensionality and the geometry of location**: knowledge and skills and performance descriptions.

(1) The student analyzes the relationship between three-dimensional objects and related two-dimensional representations and uses these representations to solve problems.	Following are performance descriptions. (A) The student describes, and draws cross sections and other slices of three-dimensional objects. (B) The student uses nets to represent and construct three-dimensional objects. (C) The student uses top, front, side, and corner views of three-dimensional objects to create accurate and complete representations and solve problems.

(2)	The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.	Following are performance descriptions. (A) The student uses one- and two-dimensional coordinate systems to represent points, lines, line segments, and figures. (B) The student uses slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons. (C) The student develops and uses formulas including distance and midpoint.

(e) **Congruence and the geometry of size**: knowledge and skills and performance descriptions.

(1)	The student extends measurement concepts to find area, perimeter, and volume in problem situations.	Following are performance descriptions. (A) The student finds areas of regular polygons and composite figures. (B) The student finds areas of sectors and arc lengths of circles using proportional reasoning. (C) The student develops, extends, and uses the Pythagorean Theorem. (D) The student finds surface areas and volumes of prisms, pyramids, spheres, cones, and cylinders in problem situations.
(2)	The student analyzes properties and describes relationships in geometric figures.	Following are performance descriptions. (A) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties of parallel and perpendicular lines. (B) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of polygons and their component parts. (C) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them. (D) The student analyzes the characteristics of three-dimensional figures and their component parts.
(3)	The student applies the concept of congruence to justify properties of figures and solve problems.	Following are performance descriptions. (A) The student uses congruence transformations to make conjectures and justify properties of geometric figures. (B) The student justifies and applies triangle congruence relationships.

(f) **Similarity and the geometry of shape**: knowledge and skills and performance descriptions.

	The student applies the concepts of similarity to justify properties of figures and solve problems.	Following are performance descriptions.

(1) The student uses similarity properties and transformations to explore and justify conjectures about geometric figures.

(2) The student uses ratios to solve problems involving similar figures.

(3) In a variety of ways, the student develops, applies, and justifies triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples.

(4) The student describes the effect on perimeter, area, and volume when length, width, or height of a three-dimensional solid is changed and applies this idea in solving problems.

Source: The provisions of this §111.34 adopted to be effective September 1, 1996, 21 TexReg 7371.

§111.35. Precalculus (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998, and at that time shall supersede §75.63(bb) of this title (relating to Mathematics). Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Introduction.

(1) In Precalculus, students continue to build on the K-8, Algebra I, Algebra II, and Geometry foundations as they expand their understanding through other mathematical experiences. Students use symbolic reasoning and analytical methods to represent mathematical situations, to express generalizations, and to study mathematical concepts and the relationships among them. Students use functions, equations, and limits as useful tools for expressing generalizations and as means for analyzing and understanding a broad variety of mathematical relationships. Students also use functions as well as symbolic reasoning to represent and connect ideas in geometry, probability, statistics, trigonometry, and calculus and to model physical situations. Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology to model functions and equations and solve real-life problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning. Students also use multiple representations, applications and modeling, justification and proof, and computation in problem-solving contexts.

(c) Knowledge and skills.

(1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, radical, exponential, logarithmic, trigonometric, and piecewise-defined functions.

The student is expected to:

(A) describe parent functions symbolically and graphically, including $y = x^n$, $y = \ln x$, $y = \log_a x$, $y = \frac{1}{x}$, $y = e^x$, $y = a^x$, $y = \sin x$, etc.;

(B) determine the domain and range of functions using graphs, tables, and symbols;

(C) describe symmetry of graphs of even and odd functions;

(D) recognize and use connections among significant points of a function (roots, maximum points, and minimum points), the graph of a function, and the symbolic representation of a function; and

(E) investigate continuity, end behavior, vertical and horizontal asymptotes, and limits and connect these characteristics to the graph of a function.

(2) The student interprets the meaning of the symbolic representations of functions and operations on functions within a context.

The student is expected to:

(A) apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, $|f(x)|$, $f(|x|)$, to the parent functions;

(B) perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; and

		(C) investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.
(3)	The student uses functions and their properties to model and solve real-life problems.	The student is expected to: (A) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data; (B) use regression to determine a function to model real-life data; (C) use properties of functions to analyze and solve problems and make predictions; and (D) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas.
(4)	The student uses sequences and series to represent, analyze, and solve real-life problems.	The student is expected to: (A) represent patterns using arithmetic and geometric sequences and series; (B) use arithmetic, geometric, and other sequences and series to solve real-life problems; (C) describe limits of sequences and apply their properties to investigate convergent and divergent series; and (D) apply sequences and series to solve problems including sums and binomial expansion.
(5)	The student uses conic sections, their properties, and parametric representations to model physical situations.	The student is expected to: (A) use conic sections to model motion, such as the graph of velocity vs. position of a pendulum and motions of planets; (B) use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound; (C) convert between parametric and rectangular forms of functions and equations to graph them; and (D) use parametric functions to simulate problems involving motion.
(6)	The student uses vectors to model physical situations.	The student is expected to: (A) use the concept of vectors to model situations defined by magnitude and direction; and (B) analyze and solve vector problems generated by real-life situations.

Source: The provisions of this §111.35 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) General requirements. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(b) Introduction.

(1) In Mathematical Models with Applications, students continue to build on the K-8 and Algebra I foundations as they expand their understanding through other mathematical experiences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, to model information, and to solve problems from various disciplines. Students use mathematical methods to model and solve real-life applied problems involving money, data, chance, patterns, music, design, and science. Students use mathematical models from algebra, geometry, probability, and statistics and connections among these to solve problems from a wide variety of advanced applications in both mathematical and nonmathematical situations. Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology to link modeling techniques and purely mathematical concepts and to solve applied problems.

(2) As students do mathematics, they continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning. Students also use multiple representations, applications and modeling, justification and proof, and computation in problem-solving contexts.

(c) Knowledge and skills.

(1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems.	The student is expected to: (A) compare and analyze various methods for solving a real-life problem; (B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and (C) select a method to solve a problem, defend the method, and justify the reasonableness of the results.
(2) The student uses graphical and numerical techniques to study patterns and analyze data.	The student is expected to: (A) interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, and scatterplots to draw conclusions from the data; (B) analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences; (C) analyze graphs from journals, newspapers, and other sources to determine the validity of stated arguments; and (D) use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information.

(3)	The student develops and implements a plan for collecting and analyzing data in order to make decisions.	The student is expected to: (A) formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions; (B) communicate methods used, analysis conducted, and conclusions drawn for a data-analysis project by written report, visual display, oral report, or multi-media presentation; and (C) determine the appropriateness of a model for making predictions from a given set of data.
(4)	The student uses probability models to describe everyday situations involving chance.	The student is expected to: (A) compare theoretical and empirical probability; and (B) use experiments to determine the reasonableness of a theoretical model such as binomial, geometric, etc.
(5)	The student uses functional relationships to solve problems related to personal income.	The student is expected to: (A) use rates, linear functions, and direct variation to solve problems involving personal finance and budgeting, including compensations and deductions; (B) solve problems involving personal taxes; and (C) analyze data to make decisions about banking.
(6)	The student uses algebraic formulas, graphs, and amortization models to solve problems involving credit.	The student is expected to: (A) analyze methods of payment available in retail purchasing and compare relative advantages and disadvantages of each option; (B) use amortization models to investigate home financing and compare buying and renting a home; and (C) use amortization models to investigate automobile financing and compare buying and leasing a vehicle.
(7)	The student uses algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.	The student is expected to: (A) analyze types of savings options involving simple and compound interest and compare relative advantages of these options; (B) analyze and compare coverage options and rates in insurance; and (C) investigate and compare investment options including stocks, bonds, annuities, and retirement plans.

(8)	The student uses algebraic and geometric models to describe situations and solve problems.	The student is expected to: (A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology; (B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion; and (C) use direct and inverse variation to describe physical laws such as Hook's, Newton's, and Boyle's laws.
(9)	The student uses algebraic and geometric models to represent patterns and structures.	The student is expected to: (A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and (B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

Source: The provisions of this §111.36 adopted to be effective September 1, 1998, 22 TexReg 7623.

Chapter 111. Texas Essential Knowledge and Skills for Mathematics

Subchapter D. Other High School Mathematics Courses

Statutory Authority: The provisions of this Subchapter D issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.51. Implementation of Texas Essential Knowledge and Skills for Mathematics, Other High School Mathematics Courses.

The provisions of this subchapter shall be implemented by school districts beginning September 1, 1998, and at that time shall supersede §75.63(o), (q)-(u), and (cc) of this title (relating to Mathematics).

Source: The provisions of this §111.51 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.52. Independent Study in Mathematics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of Independent Study in Mathematics. Required prerequisites: Algebra II, Geometry. Students may repeat this course with different course content for a second credit.

(b) Content requirements. Students will extend their mathematical understanding beyond the Algebra II level in a specific area or areas of mathematics, such as theory of equations, number theory, non-Euclidean geometry, advanced survey of mathematics, or history of mathematics. The requirements for each course must be approved by the local district before the course begins.

(c) If this course is being used to satisfy requirements for the Distinguished Achievement Program, student research/products must be presented before a panel of professionals or approved by the student's mentor.

Source: The provisions of this §111.52 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.53. Advanced Placement (AP) Statistics (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for Advanced Placement (AP) Statistics are prescribed in the College Board Publication Advanced Placement Course Description: Statistics, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.53 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.54. Advanced Placement (AP) Calculus AB (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus AB are prescribed in the College Board Publication Advanced Placement Course Description Mathematics: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.54 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.55. Advanced Placement (AP) Calculus BC (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Precalculus.

(b) Content requirements. Content requirements for Advanced Placement (AP) Calculus BC are prescribed in the College Board Publication Advanced Placement Course Description: Calculus AB, Calculus BC, published by The College Board. This publication may be obtained from the College Board Advanced Placement Program.

Source: The provisions of this §111.55 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.56. IB Mathematical Studies Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Studies Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Studies Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.56 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.57. IB Mathematical Methods Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematical Methods Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisites: Algebra II, Geometry.

(b) Content requirements. Content requirements for IB Mathematical Methods Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.57 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.58. IB Mathematics Higher Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Mathematics Higher Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematical Studies Subsidiary Level or IB Mathematical Methods Subsidiary Level.

(b) Content requirements. Content requirements for IB Mathematics Higher Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.58 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.59. IB Advanced Mathematics Subsidiary Level (One-Half to One Credit).

(a) General requirements. Students can be awarded one-half to one credit for successful completion of IB Advanced Mathematics Subsidiary Level. To offer this course, the district must meet all requirements of the International Baccalaureate Organization, including teacher training/certification and IB assessment. Recommended prerequisite: IB Mathematics Higher Level.

(b) Content requirements. Content requirements for IB Advanced Mathematics Subsidiary Level are prescribed by the International Baccalaureate Organization. Curriculum guides may be obtained from International Baccalaureate of North America.

Source: The provisions of this §111.59 adopted to be effective September 1, 1998, 22 TexReg 7623.

§111.60. Concurrent Enrollment in College Courses.

(a) General requirements. Students shall be awarded one-half credit for each semester of successful completion of a college course in which the student is concurrently enrolled while in high school.

(b) Content requirements. In order for students to receive state graduation credit for concurrent enrollment courses, content requirements must meet or exceed the essential knowledge and skills in a given course

Source: The provisions of this §111.60 adopted to be effective September 1, 1998, 22 TexReg 7623